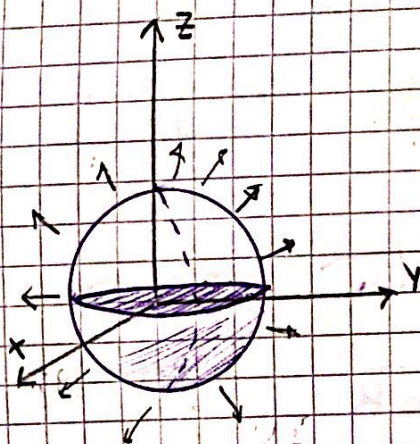


GUÍA 2: ELECTROESTÁTICA EN CONDUCTORES Y DIELECTRICOS

1

ESFERA METALICA MACIZA DE RADIO R CON CARGA Q

ES CONDUCTORA ENTONCES LA CARGA ES SUPERFICIAL (σ_0)



USANDO GAUSS

CON $R < A$

$$\iint_{\pi \cdot 2\pi} \vec{E} \cdot d\vec{S} = \frac{Q_{ENC}}{\epsilon_0}$$

$$E \int_0^{2\pi} \int_0^\pi R^2 \sin \theta \, d\psi \, d\theta = \frac{Q_{ENC}}{\epsilon_0}$$

$$E \cdot 4\pi R^2 = \frac{Q}{\epsilon_0} \longrightarrow E = \frac{Q}{4\pi R^2 \epsilon_0}$$

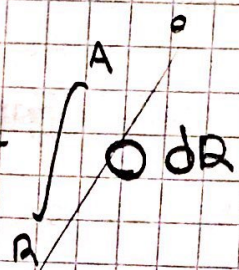
$$E(\vec{R}) \begin{cases} 0 & \text{si } R > A \\ \frac{Q}{4\pi R^2 \epsilon_0} & \text{si } R < A \end{cases}$$

$$V(\infty) = 0$$

$$\Delta V = - \int_{\infty}^R \vec{E} \cdot d\vec{l} = - \int_{\infty}^R \frac{Q}{4\pi R^2 \epsilon_0} dR = - \frac{Q}{4\pi \epsilon_0} \int_{\infty}^R \frac{1}{R^2} dR =$$

$$\Delta V = - \frac{Q}{4\pi \epsilon_0} \left(-\frac{1}{R} \right) = \frac{Q}{4\pi \epsilon_0 R}$$

$R > A$

$$\Delta V = - \left[\int_{\infty}^R \frac{Q}{4\pi \epsilon_0 R^2} dR + \int_A^R 0 dR \right] = \frac{Q}{4\pi \epsilon_0 R}$$


$$\Delta V \text{ (EN TODO EL ESPACIO)} = \frac{Q}{4\pi \epsilon_0 R}$$

$$R = 2R \quad V(2R) = 0$$

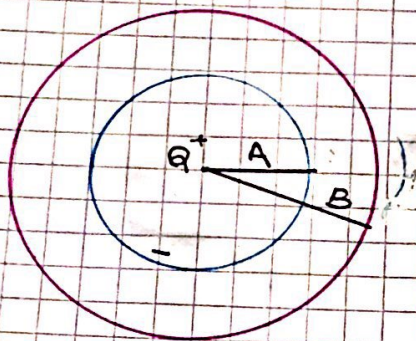
$$\Delta V = - \int_{2R}^R \frac{Q}{4\pi \epsilon_0 R^2} dR = \frac{Q}{4\pi \epsilon_0 R} \Big|_{2R}^R = \frac{Q}{4\pi \epsilon_0} \left(\frac{1}{R} - \frac{1}{2R} \right)$$

$$W = \int_{2R+\infty}^{\infty} -\vec{q} \cdot \vec{E} dL = \int_{2R}^{\infty} \frac{Q}{4\pi R^2 \epsilon_0} dR = \frac{Qq}{4\pi 2R \epsilon_0}$$

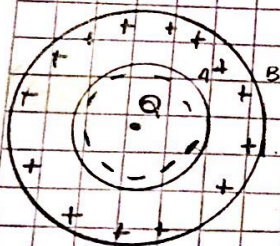
↓ MISMO SENTIDO QUE EL CAMPO

2-

UNA CASCA DA CONDUCTORA ESFERICA



A CASCA DA DESCARGADA



$$E(\vec{R}) \begin{cases} 0 & \text{si } A < R < B \\ \frac{Q}{4\pi\epsilon_0 R^2} & \text{si } R > B \\ \frac{Q}{4\pi\epsilon_0 R^2} & \text{si } R < A \end{cases}$$

AFUEBA DEL CONDUCTOR ($R > B$)

$$\Delta V = - \int_{\infty}^R \frac{Q}{4\pi\epsilon_0 R^2} dR = \frac{Q}{4\pi\epsilon_0 R}$$

DENTRO DEL CONDUCTOR ($A < R < B$)

$$\Delta V = V(A) - V(\infty) = - \left[\int_{\infty}^R \frac{Q}{4\pi\epsilon_0 R^2} dR + \int_B^A 0 dR \right] = \frac{Q}{4\pi\epsilon_0 R}$$

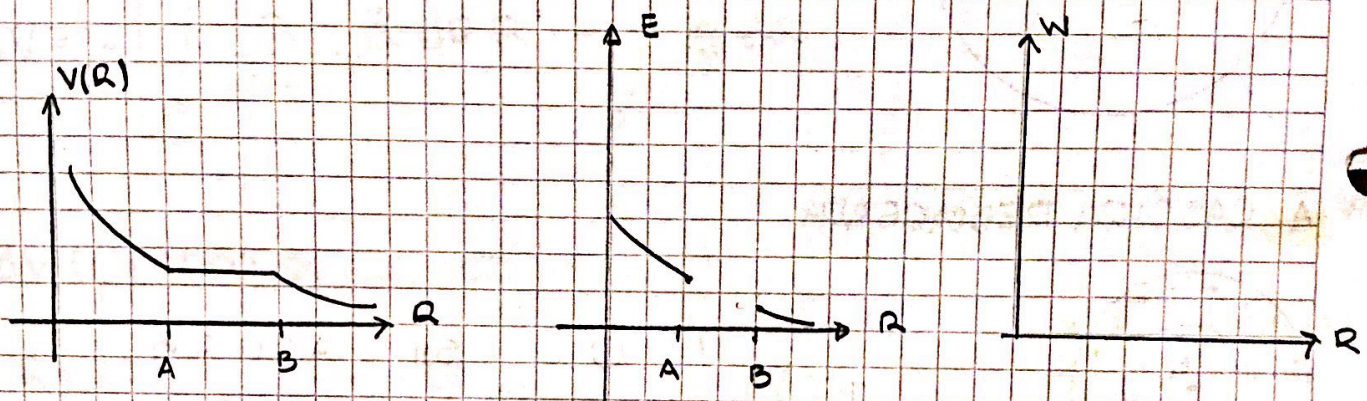
EN EL HUECO ($R < A$)

$$\Delta V = V(R) - V(\infty) = \int_{\infty}^B \frac{Q}{4\pi\epsilon_0 R^2} dR + \int_B^A 0 dR + \int_A^R \frac{Q}{4\pi\epsilon_0 R^2} dR$$

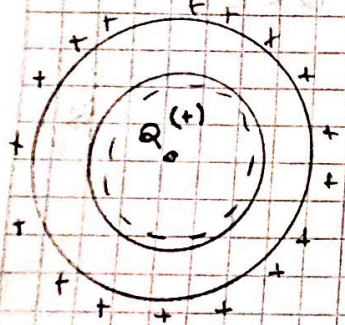
NOTA $\Delta V = - \left[\frac{-Q}{4\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{A} \right) \right] \Rightarrow \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} + \left(\frac{1}{R} - \frac{1}{A} \right) \right)$

CALCULAR EL W PARA LLEVAR A Q_0 ENTRE DOS PUNTOS.

$$W = q \Delta V$$



B. CASCARA CARGADA CON $Q = -3 \mu C$



$$\text{SUP A} = -1 \mu C \quad (-q)$$

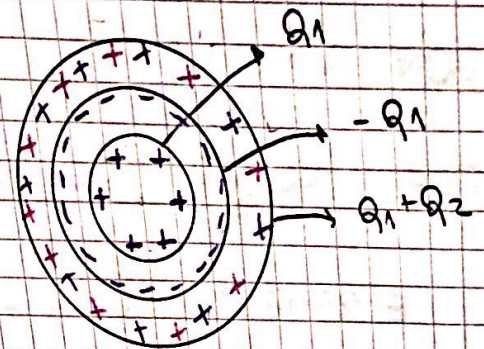
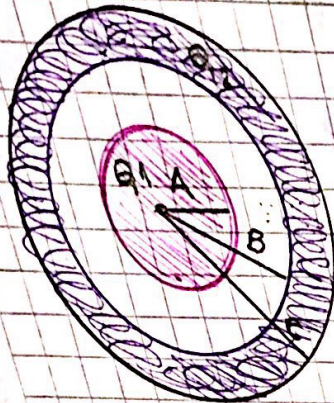
$$\text{SUP B} = 1 \mu C + -3 \mu C = -2 \mu C \quad (q + Q)$$

$$\text{EL CAMPO } E = \begin{cases} \frac{q}{4\pi\epsilon_0 R^2} \vec{R} & \text{si } R < A \\ 0 & \text{si } A < R < B \\ \frac{q + Q}{4\pi\epsilon_0 R^2} & \text{si } R > B \end{cases}$$

$\Delta V =$ TODO LO MISMO QUE EN EL 2A.

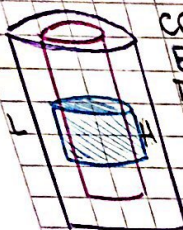
3-

CONDUCTOR CILINDRICO \rightarrow L (LARGO)
 \searrow
 A (RADIO)



A- LAS CARGAS SE DISTRIBUYEN SUPERFICIALMENTE

B- CALCULAR σ_A , σ_B , σ_C Y EL CAMPO



COMO H (GAUSSEANA) \ll L (CONDUCTOR) PUEDO
 DESPRECIAR LOS EFECTOS DE BORDE. DE ESTA MANERA
 PUEDO DECIR QUE LA DIRECCION DEL CAMPO ES RADIAL
 YA QUE HAY SIMETRIA DE ROTACION Y MOV.

4 ZONAS PARA CALCULAR EL CAMPO

$R < A \rightarrow E = 0$ (NO HAY CARGA ENCERRADA)

CON $A < R < B$ (TRABAJAR EN CILINDRICAS)

$$E \iint ds = \frac{Q_{ENC}}{\epsilon_0} \rightarrow E \left(\iint_{LAT} ds + \iint_{TAPA} ds + \iint_{TAPA} ds \right) = \frac{Q_{ENC}}{\epsilon_0}$$

$$E \int_0^H \int_0^{2\pi} R d\phi dz = \frac{\sigma_A \cdot 2\pi R A H}{\epsilon_0} = E R \int_0^H \int_0^{2\pi} d\phi dz = \frac{\sigma_A 2\pi R A H}{\epsilon_0}$$

$$E = \frac{\sigma_A R A}{\epsilon_0 R}$$

NOTA

CON $B < R < C$

$E = 0$ → POR DEF DE CONDUCTOR

$$Q_{ENC} = 0 \rightarrow \sigma_A R_A 2\pi H + \sigma_B R_B 2\pi H = 0$$

$$\sigma_A R_A / 2\pi = -\sigma_B R_B / 2\pi H$$

$$\sigma_A = -\frac{\sigma_B R_B}{R_A}$$

CON $R > C$

$$E 2\pi R H = \frac{Q_1 + Q_2}{\epsilon_0}$$

$$E 2\pi R H = \frac{\sigma_A R_A 2\pi H + \sigma_B R_B 2\pi H + \sigma_C R_C 2\pi H}{\epsilon_0}$$

$$E 2\pi R H = \frac{\sigma_C R_C 2\pi H}{\epsilon_0} \rightarrow E = \frac{\sigma_C R_C}{\epsilon_0 R}$$

$$Q_1 = \sigma_A 2\pi L R_A$$

$$\sigma_A = \frac{Q_1}{2\pi R_A L} = -\frac{\sigma_B R_B}{R_A} \rightarrow \frac{Q_1 R_A}{2\pi R_A L R_B} = \sigma_B$$

$$\sigma_B = -\frac{Q_1}{2\pi L R_B}$$

$$\sigma_C = \frac{Q_2 + Q_1}{2\pi R_C L}$$

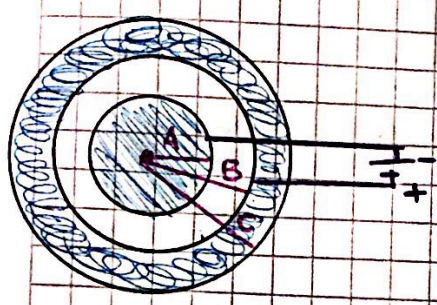
D - $V(C) - V(A) = -\int_{R_A}^{R_B} \frac{\sigma_A R_A}{\epsilon_0 R} dr + \int_{R_B}^{R_C} 0 dr = -\frac{\sigma_A R_A}{\epsilon_0} \ln\left(\frac{R_B}{R_A}\right)$ (NO HAY CAMPO)

$$V(B) - V(A) = -\int_{R_A}^{R_B} \frac{\sigma_A R_A}{\epsilon_0 R} dr = -\frac{\sigma_A R_A}{\epsilon_0} (\ln(R_B) - \ln(R_A))$$

$$V(R) - V(R_A) = (\text{HAGO POR PARTES COMO SIEMPRE})$$

NOTA

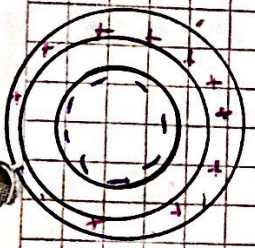
4



$$V(B) - V(A) = 10 \text{ V}$$



CUANDO VOY DE
 $\oplus A \ominus \rightarrow \Delta V < 0$
 $\ominus A \oplus \rightarrow \Delta V > 0$



$$Q_A = -Q_B$$

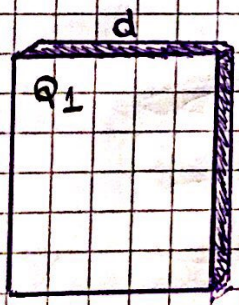
$$\sigma_A \neq \sigma_B \quad \sigma_C = 0$$

↓
 POR ESO CONECTO
 ASI LA PILA

CUENTAS HECHAS EN CLASE !

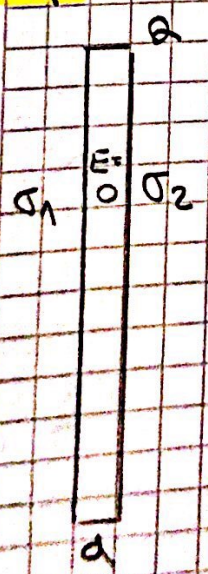
NOVEDOSO $R > C \rightarrow E = 0$ XQ NO HAY CARGAS EN σ_C
 Y LAS DE A Y B SE COMPENSAN

5



PLACA CONDUCTORA (CON ESPESOR d)
 $L \gg d$. LO TRABAJA COMO PLANO ∞

A- CALCULAR LAS DENSIDADES DE CARGA LIBRE



$$Q = \sigma_1 S + \sigma_2 S$$

COMO $L \gg d$ DESPRECIO LAS
 CONDICIONES DE BORDE Y TRABAJA
 COMO UN PLANO ∞

7-

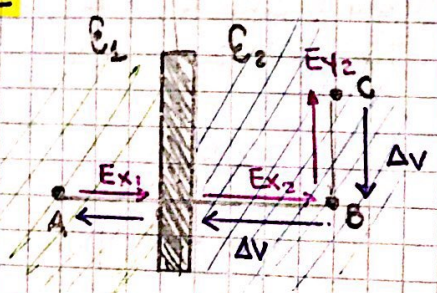
$$D = \epsilon \bar{E} = \epsilon_0 \epsilon_r \bar{E}$$

ESTO SE CUMPLE EN MATERIALES LINEALES, HOMOGENEO E ISOTOPOS.

$$D = \epsilon \bar{E}$$

$$\bar{D} = \epsilon_0 \bar{E} + P$$

8-



$V_A - V_B = 200V \rightarrow V$ CRECE DE BAA
 $V_B - V_C = 50V \rightarrow V$ CRECE DE CAB
 E ES UNIFORME.

HALLAR E, D Y P.

AO = 10cm BO = 20cm BC = 5cm

$$\Delta V = V_A - V_B = - \int_{0,2}^0 E_{x_2} dx + - \int_0^{-0,1} E_{x_1} dx = 200V$$

↳ TENGO QUE IR DE BAA

COMO LOS CAMPOS SON UNIFORMES, PUEDO SACARLO AFUERA DE LA INTEGRAL

$$- E_{x_2} \int_{0,2}^0 dx + - E_{x_1} \int_0^{-0,1} dx$$

$$- E_{x_2} (0 - 0,2) + - E_{x_1} (-0,1 - 0) = 200V$$

↳ DOS INCOGNITAS, 1 ECUACION

CONDICIÓN DE FRONTERA

$$D_{1N} = D_{2N} \rightarrow D = \epsilon \bar{E}$$

1) COMO NO LIBRE, PUEDO DECIR QUE

$$\epsilon_0 \epsilon_1 E_{1x} = \epsilon_0 \epsilon_2 E_{2x}$$

$$\left. \begin{matrix} \epsilon_{r1} = 3,5 \\ \epsilon_{r2} = 6,25 \end{matrix} \right\} \begin{matrix} 3,5 E_{1x} = 6,25 E_{2x} \\ E_{1x} = 1,79 E_{2x} \end{matrix}$$

$$-E_{x_2}(-0,2) + -1,79 E_{2x}(-0,1) = 200 \text{ V}$$

$$0,2 E_{x_2} + 0,180 E_{2x} = 200 \text{ V}$$

$$0,38 E_{2x} = 200$$

$$\bullet E_{2x} = 526,3 \text{ V/M}$$

$$\bullet E_{1x} = 1,79 E_{2x} = 1,79 \cdot 526,3 = 942,1 \text{ V/M}$$

$$2) E_{1y} = E_{2y}$$

LAS COMPONENTES EN Y SON IGUALES

$$\Delta V = V_B - V_C = - \int_{0,05}^0 E_{y_2} dy = -E_{y_2} (0 - 0,05) = 50 \text{ V}$$

$$0,05 E_{y_2} = 50 \text{ V}$$

$$\bullet E_{y_2} = 1000 \text{ V/M} \quad E_{y_1} = 1000 \text{ V/M}$$

$$E_1 = 942 \text{ V/M } \hat{i} + 1000 \text{ V/M } \hat{j} \quad |E_1| = 1373,8 \text{ V/M}$$

$$E_2 = 527 \text{ V/M } \hat{i} + 1000 \text{ V/M } \hat{j} \quad |E_2| = 1130,4 \text{ V/M}$$

- CALCULO \vec{D}

COMO DIJIMOS $\rightarrow D = \epsilon_0 \epsilon_r E$

$$D_1 = \epsilon_0 \epsilon_{r_1} E_1 = 4,3 \times 10^{-8} \text{ V/M}$$

$$D_2 = \epsilon_0 \epsilon_{r_2} E_2 = 6,25 \times 10^{-8} \text{ V/M}$$

- CALCULO \vec{P}

PUEDO DECIR $P = \epsilon_0 (\epsilon_r - 1) E$

$$\vec{P} = D - \epsilon_0 \vec{E}$$

$$P_1 = 3,08 \times 10^{-8} \text{ V/M}$$

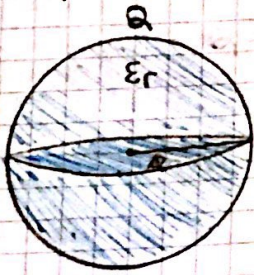
$$P_2 = 5,25 \times 10^{-8} \text{ V/M}$$

9-

$$Q = 2\rho C (\text{UNIFORME } \rho)$$

$$R = 10 \text{ cm}$$

$\epsilon_r = 2,8$
DIELECTRICA



A- CALCULAR EL CAMPO \vec{E} Y ΔV DESDE EL ∞ .

1) CALCULO \vec{D}

UTILIZO GAUSS GENERALIZADO $\iint \vec{D} \cdot d\vec{s} = Q_L^{ENC}$

• DIRECCIÓN DE \vec{D} = RADIAL (\hat{r})

• DEPENDENCIA DE \vec{D} = RADIAL (\hat{r}) YA QUE SI LA GIRO EN θ O ψ NO NOTO DIFERENCIA EN \vec{D}
↳ EJE SIMETRÍA

• $R < A$

$$\iint \vec{D} \cdot d\vec{s} = Q_L \rightarrow D \iint ds = Q_L$$

$$D \int_0^{2\pi} \int_0^\pi R^2 \sin\theta \, d\theta \, d\psi = Q_L$$

$$D R^2 4\pi = Q_L^{ENC} \rightarrow D R^2 4\pi = \rho \cdot \frac{4}{3} \pi R^3$$

$$D = \frac{\rho}{3} \frac{\pi R^3}{4\pi R^2}$$

$$D = \rho \frac{R}{3} \hat{r}$$

• $R > A$

$$\iint \vec{D} \cdot d\vec{s} = Q_L \rightarrow D R^2 4\pi = \rho \frac{4}{3} \pi A^3$$

$$D = \rho \frac{A^3}{3R^2} \hat{r}$$

NOTA

$$\vec{D} \begin{cases} \rho \frac{R}{3} \hat{r} & \text{si } R < A \\ \rho \frac{A^3}{3R^2} \hat{r} & \text{si } R > A \end{cases}$$

2) CALCULO \vec{E}

$$\text{ADENTRO} = \epsilon_0 \epsilon_r \vec{E} = D$$

$$\text{AFUEIRA} = \epsilon_0 \vec{E} = D$$

$$\vec{E} \begin{cases} \rho \frac{R}{3\epsilon_0 \epsilon_r} \hat{r} & \text{si } R < A \\ \rho \frac{A^3}{3R^2 \epsilon_0} \hat{r} & \text{si } R > A \end{cases}$$

$$\Delta V = - \int E dl \quad V(\infty) = 0 \text{ REF}$$

CON $R > A$

$$\Delta V = V(R) - V(\infty) = - \int_{\infty}^R \rho \frac{A^3}{3R^2 \epsilon_0} \hat{r} \frac{1}{R} dR \hat{r} =$$

$$- \rho \frac{A^3}{3\epsilon_0} \int_{\infty}^R \frac{1}{R^2} dR = \frac{1}{R} \Big|_{\infty}^R = \frac{1}{R} - 0$$

$$\Delta V = - \rho \frac{A^3}{3\epsilon_0 R}$$

CON $R < A$

$$\Delta V = V(R) - V(\infty) = - \int_{\infty}^A \rho \frac{A^3}{3R^2 \epsilon_0} dr + - \int_A^R \rho \frac{R}{3\epsilon} dr$$

$$\Delta V = - \rho \frac{A^3}{3\epsilon_0} \int_{\infty}^A \frac{1}{R^2} dR + - \rho \frac{1}{3\epsilon} \int_A^R R dR$$

$$- \rho \frac{A^3}{3\epsilon_0} \cdot \frac{1}{A} + - \rho \frac{1}{3\epsilon} \cdot \frac{R^2}{2} \Big|_A^R = - \rho \frac{1}{3\epsilon} \left(\frac{R^2}{2} - \frac{A^2}{2} \right)$$

$$\Delta V = \left(- \int \frac{\rho}{3\epsilon_0} \right) + \left(- \int \frac{1}{\epsilon_0 \epsilon_r} \right) \left(\frac{R^2}{2} - \frac{A^2}{2} \right)$$

$$\Delta V = - \int \frac{\rho}{\epsilon_0} \left(\frac{A^2}{3} + \frac{1}{\epsilon_r} \left(\frac{R^2}{2} - \frac{A^2}{2} \right) \right)$$

B- CALCULAR $\bar{\sigma}_p$ Y \int_p

CALCULO \bar{P} : $\bar{P} = \bar{D} - \epsilon_0 \bar{E}$

$$\bar{P} = \int \frac{\rho}{3} - \epsilon_0 \left(\int \frac{\rho}{3\epsilon_0 \epsilon_r} \right)$$

$$\bar{P} = \int \frac{\rho}{3} - \int \frac{\rho}{3\epsilon_r} = \int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right)$$

$$\bar{P} \begin{cases} \int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right) & \text{SII } R < A \\ 0 & \text{SII } R > A \end{cases}$$

CONDICIONES DE BORDE 1) $D_{1N} - D_{2N} = \sigma_L$

EVALUO AMBAS EXPRESIONES EN $R=A$

$$\int \frac{\rho}{3} - \int \frac{\rho}{3A^2} = \sigma_L$$

$0 = \sigma_L$ NO HAY CARGA LIBRE

• $\sigma_p = \bar{P} \cdot \hat{N} \rightarrow$ EVALUO P EN $R=A$ Y TOMO \hat{R} (SENTIDO DE \bar{E})

$$\sigma_p = \int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right) \hat{R}$$

• $\int_p = -\bar{\nabla} P$ (EN ESFERICAS)

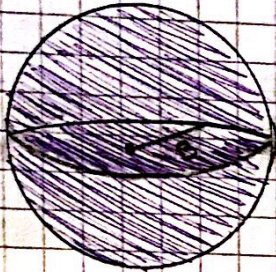
$$\bar{\nabla} P = \frac{1}{R^2} \frac{d}{dR} (R^2 P_r) = \frac{1}{R^2} R^2 \left(\int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right) \right)$$

NOTA $\bar{\nabla} P = \rho \cdot \frac{1}{3} \left(1 - \frac{1}{\epsilon_r} \right) = \frac{1}{R^2} \int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right) = \int \frac{\rho}{3} \left(1 - \frac{1}{\epsilon_r} \right)$

10

SI SOLO FUESE SUPERFICIAL (σ) NO HABRIA E, D, P A DENTRO

11



$$\rho(r) = Ar$$

A- CALCULAR EL CAMPO \vec{E}

$$\text{GAUSS GENERALIZADO} \rightarrow \iint D \, ds = Q_L$$

CON $R < B$

$$D \cdot 4\pi R^2 = Q_L^{\text{ENC}}$$

$$D \cdot 4\pi R^2 = \iiint \rho \, dv$$

$$D \cdot 4\pi R^2 = \int_0^R \int_0^{2\pi} \int_0^\pi R^2 \sin\theta \cdot AR \, d\theta \, d\varphi \, dr$$

$$A \int_0^R \int_0^{2\pi} \int_0^\pi R^3 \sin\theta \, d\theta \, d\varphi \, dr = \int_0^R R^3 \cdot 2\pi \, dr = \int_0^R R^3 4\pi \, dr$$

$$\frac{R^4 4\pi A}{4} \rightarrow AR^4 \pi = D 4\pi R^2$$

$$D = \frac{AR^2}{4} \rightarrow E = \frac{AR^2}{4\epsilon_0 \epsilon_r} \hat{r}$$

CON $R > B$

$$D 4\pi R^2 = AB^4 \pi \rightarrow D = \frac{AB^4}{4R^2} \rightarrow E = \frac{AB^4}{4R^2 \epsilon_0} \hat{r}$$

NOTA

$$\Delta V = - \int \vec{E} \cdot d\vec{l}$$

$$\boxed{R > B}$$

$$\Delta V = V(B) - V(\infty) = - \int_{\infty}^B \frac{AB^4}{4R^2\epsilon_0} dR = - \frac{AB^3}{4\epsilon_0}$$

$$\boxed{R < B}$$

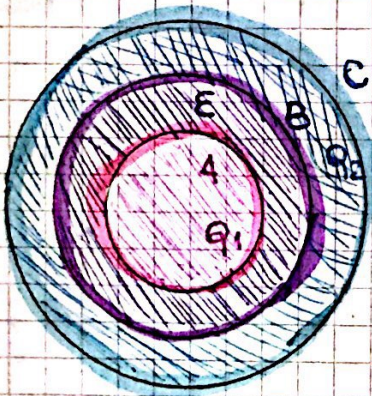
$$\Delta V = V(R) - V(\infty) = - \int_{\infty}^B \frac{AB^4}{4R^2\epsilon_0} dR + - \int_B^R \frac{AR^2}{4\epsilon_0\epsilon_r} dR$$

$$\Delta V = - \frac{AB^3}{4\epsilon_0} + \left(- \left(\frac{AR^3}{12\epsilon_0\epsilon_r} - \frac{AB^3}{12\epsilon_0\epsilon_r} \right) \right)$$

$$\Delta V = - \frac{AB^3}{4\epsilon_0} + \left(- \frac{AR^3}{12\epsilon} + \frac{AB^3}{12\epsilon} \right)$$

12

CON EJ 3



CONDUCTOR - DIELECTRICO - CONDUCTOR

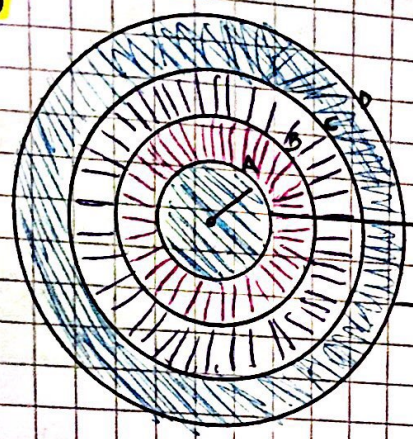
A - LAS CARGAS EN EL CONDUCTOR

SON SUPERFICIALES COMO HABIAMOS

DICHO, AHORA LAS CARGAS EN EL DIELECT

SE FORMAN DIPOLOS EN TODO EL VOLUMEN

B - CALCULAR DENSIDAD DE CARGA



CONDUCTORES INICIALMENTE
DESCARGADOS

$$\left. \begin{matrix} \frac{1}{r} \\ \frac{1}{r} \end{matrix} \right\} \sigma \text{ SE COMPENSA} \\ V_0 \quad L \gg d$$

A DISTRIBUCION DE CARGA EN LOS CONDUCTORES
COMO SABEMOS EL CAMPO \vec{E} EN UN CONDUCTOR ES
CERO.

EL CAMPO ENTRE C Y D $\rightarrow E = 0$ LAS CARGAS
TMB COMO EL CONDUCTOR ESTA INICIALMENTE SE COMPENSAN
DESCARGADO $Q_i = Q_f = 0$ ENTONCES

$$Q_A + Q_C = 0 \rightarrow Q_C = -Q_A$$

LUEGO AL ENCENDIDO TODO EL CONDUCTOR CON UNA GAUSSIANA
LA Q_T TIENE QUE SER CERO, ENTONCES

$$Q_A + Q_C + Q_D = 0$$

$$Q_A - Q_A + Q_D = 0 \rightarrow Q_D = 0$$

$\iint D \cdot ds = Q_L$ \rightarrow HAGO GAUSS GENERALIZADO, USO COMO
SUP UN CILINDRO TMB. COMO $L \gg d$

($\perp A \vec{D}$) DESPRECIO EFECTOS DE BORDE

$$\iint_{T_1} \vec{D} \cdot d\vec{s} + \iint_{T_2} \vec{D} \cdot d\vec{s} + \iint_{LAT} \vec{D} \cdot d\vec{s} = Q_L$$

$$\iint \vec{D} \cdot d\vec{s} = Q_L \rightarrow D \iint d\vec{s} = Q_L$$

$$D \cdot 2\pi R H = \sigma_A R A 2\pi H$$

CON $A < R < B$ CON $R < A$ → $D = 0$

$$\bar{D} = \frac{\sigma_A R A}{R}$$

CON $B < R < C$ → ES LA MISMA INTEGRAL:

$$D = \frac{\sigma_A \cdot R B}{R}$$

CON $C < R < D$ → $D = 0$ $R > D$ → $D = 0$

$$\bar{D} = \begin{cases} \frac{\sigma_A R A}{R} (\checkmark) & A < R < B & \text{COMO ES UN MATERIAL} \\ & & \text{MLIH} \\ 0 & R < A \vee C < R < D & D = \epsilon_0 \epsilon_r \bar{E} \\ & R > D \\ \frac{\sigma_A R B}{R} & \text{CON } B < R < C \end{cases}$$

$$E = \begin{cases} \frac{\sigma_A R A}{\epsilon_0 \epsilon_1 R} (\checkmark) & A < R < B \\ \frac{\sigma_A R B}{\epsilon_0 \epsilon_2 R} (\checkmark) & B < R < C \\ 0 & \text{EN OTRO CASO} \end{cases} \left\{ \begin{array}{l} \text{CAMBIO EN} \\ \text{LOS DIELECTRICOS} \end{array} \right.$$

$$P = \epsilon_0 (\epsilon_r - 1) E \left\{ \begin{array}{l} \frac{\sigma_A R A}{\epsilon_0 \epsilon_1 R} \epsilon_0 (\epsilon_1 - 1) \\ \frac{\sigma_A R B}{\epsilon_0 \epsilon_2 R} \epsilon_0 (\epsilon_2 - 1) \end{array} \right. \quad \begin{array}{l} \text{EN OTRO} \\ \text{CASO} \end{array}$$

COMO ME DIERON VO PUEDO SACAR LAS σ .

$$\Delta V = - \int E dl$$

$$\Delta V = V_0 = V(D) - V(A) = - \int_A^B \vec{E}_1 \cdot d\vec{l} + - \int_B^C \vec{E}_2 \cdot d\vec{l} + 0$$

$$- \int_A^B \frac{\sigma_A B A}{\epsilon_0 \epsilon_1 R} dR = - \frac{\sigma_A B A}{\epsilon_0 \epsilon_1} \int \frac{1}{R} dR = \ln \left(\frac{A}{B} \right)$$

$$- \int_B^C \frac{\sigma_A B A}{\epsilon_0 \epsilon_2 R} dR = - \frac{\sigma_A B A}{\epsilon_0 \epsilon_2} \int \frac{1}{R} dR = \ln \left(\frac{C}{B} \right)$$

$$- \frac{\sigma_A B A}{\epsilon_0} \left(\frac{1}{\epsilon_1} \ln \left(\frac{A}{B} \right) + \frac{1}{\epsilon_2} \ln \left(\frac{C}{B} \right) \right) = V_0$$

$$\sigma_A = - \frac{V_0 \epsilon_0}{B A} \left[\frac{1}{\epsilon_1 \ln \left(\frac{A}{B} \right) + \frac{1}{\epsilon_2 \ln \left(\frac{C}{B} \right)} \right]$$

$\sigma_C =$) LO MISMO PERO POSITIVO

$\sigma_B = 0$ (ES DE POLARIZACIÓN)

$\sigma_D = 0$